GOVERNMENT OF TAMILNADU

Department Of Employment and Training

Course: SSC

Subject: Quantitative Aptitude

Topic: Geometry

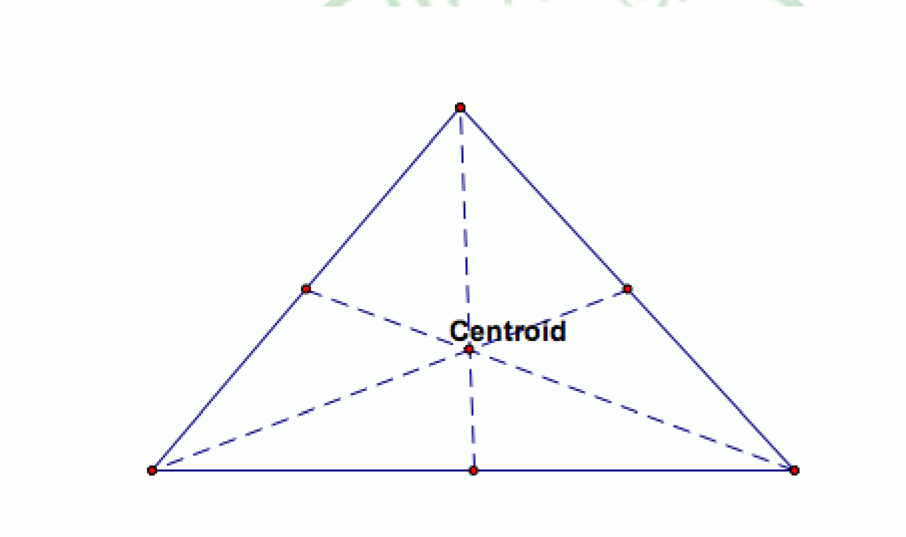
GEOMETRY:

Triangles and its various kinds of centres:

Triangle have four kind of centre namely CENTROID,ORTHOCENTER,CIRCUMCENTER and INCENTER.

Centroid:

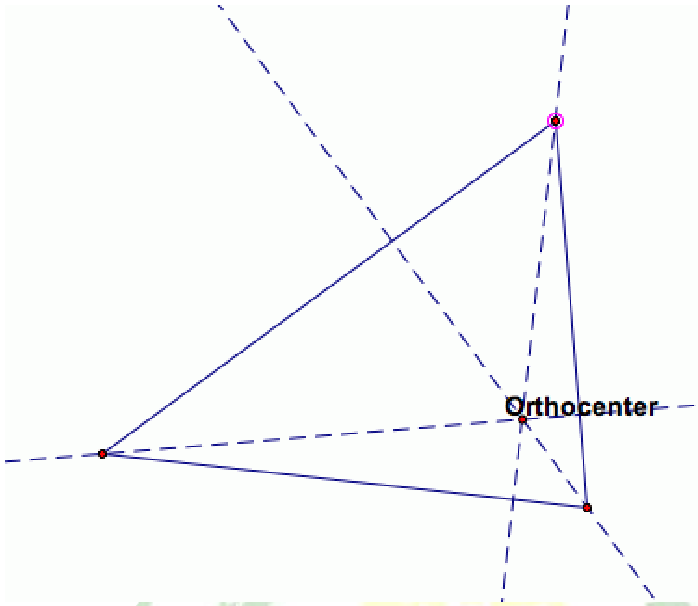
The centroid is the first center and is obtained by locating the intersection of the three medians of the triangle.The median of the triangle is obtained by joining each vertex with the midpoint of the opposite side.



- It is the point of intersection of the three median of the triangle.
- A centroid divides the area of the triangle in exactly three parts.
- A line segment joining the midpoint of the side with the opposite vertex is called median.

Orthocenter:

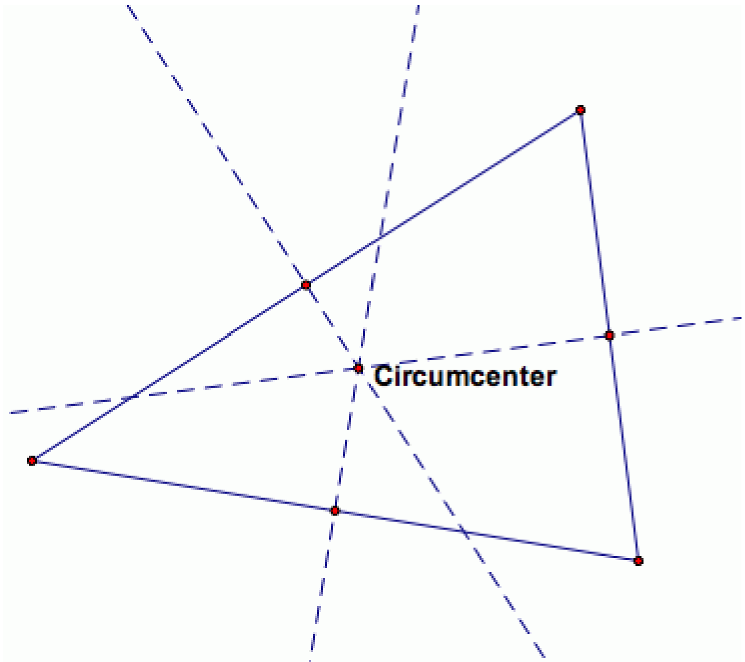
The second center of a triangle is the orthocenter.It is obtained by finding the intersection of the 3 altitudes of the triangle.An altitude is found by joining each vertex with the point on the opposite side that creates a perpendicular line with the opposite side.



- It is the point of intersection of all the 3 altitudes of the triangle.

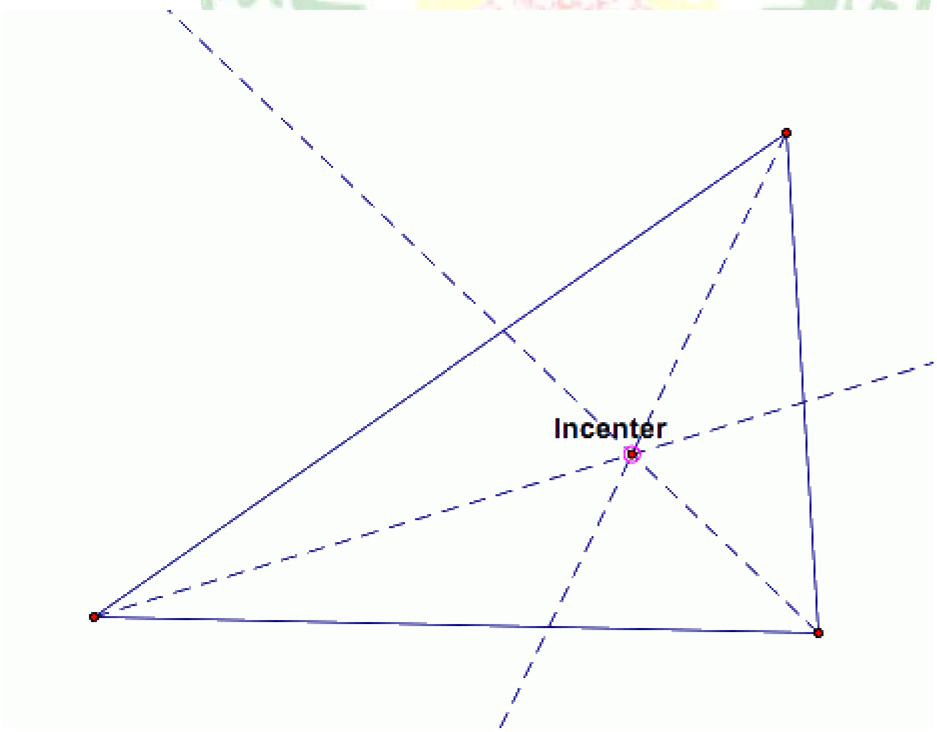
Circumcenter:

The circumcenter is the intersection of the perpendicular bisectors of each side of the triangle.



Incenter:

The last center is the incenter.The incenter is found by first constructing the angle bisectors of each of the three angles.



Congruence and similarity of triangles:

Congruence of Triangles

congruent objects have same shape and size. In the same way, two triangles are said to be congruent if their corresponding sides measure same length and eventually their corresponding angles would be the same.

Symbolically, two triangles can represented congruent by placing the sign ≅between the names of triangle. Let us suppose that two random triangles △ABC and △PQR are congruent, then we write

$$\triangle ABC \cong \triangle PQR$$

Which is read as triangle ABC is congruent to triangle PQR and vice versa.

In the case, when two triangles are congruent, it implies that their corresponding parts are equal. Thus, without even seeing a diagram, we can say that
If $\triangle ABC \cong \triangle PQR$

then AB = PQ, BC = QR, AC = PR and ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R

This property is known as "corresponding parts of congruent triangles" or "CPCT"

Rules for Congruence

Rule 1 : SSS (Side, Side, Side)
Two triangles are said to be congruent if all the three sides of a triangle are equal to the corresponding sides of another. This rule is thus known as side, side, side or in short SSS principle.

Rule 2 : SAS (Side, Angle, Side)
Two triangles can be proved congruent if two sides of a triangle are equal to corresponding sides of another and angle between them is also of same measure. For this criterion, It is mandatory that the angle to be considered is the angle **between** the equal sides. This rule is known as side, angle, side or SAS rule.

Rule 3 : AAS (Angle, Angle, Side)
If in two triangles, two corresponding angles are equal in measure and one corresponding side is equal in length, then these triangles are said to be congruent to each other. This principle is known as angle, angle, side or AAS principle.

Rule 4 : RHS (Right angle, Hypotenuse, Side)
This rule applies to the congruence of two right-angled triangles. If two triangles are right angled and having same length of hypotenuse along with another same corresponding side, then such triangles will be congruent to each other. This is known as right angle, hypotenuse, side or RHS rule.

Similarity of Triangles

Two triangles are called similar to each other, if their corresponding sides are in same ratio and all three angles are same; i.e. their shape is same and size may be different.

When two triangles are similar, we represent it by placing ∼ sign between the names of triangles. For example -

$$\triangle ABC \sim \triangle PQR$$

denotes that triangle ABC is similar to triangle PQR and vice versa.

In this case, we can conclude that

$ABPQ = BCQR = ACPR$ and $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Rules of Similarity

Rule 1 : AA (Angle, Angle)

If two angles in one triangle are equal to the corresponding angles in another triangle, then both triangles would be similar to each other. This is called angle, angle or AA criterion.

Rule 2 : SSS (Side, Side, Side)

If all three corresponding sides of two triangles are in same proportion, then they are said to be similar triangles. Thus, it is named as side, side, side or SSS principle.

Rule 3 : SAS (Side, Angle, Side)

Two triangles can be proved similar when their two corresponding sides are in same proportion and corresponding angles in between the sides are of same measure. Therefore, this is called side, angle side or SAS rule.

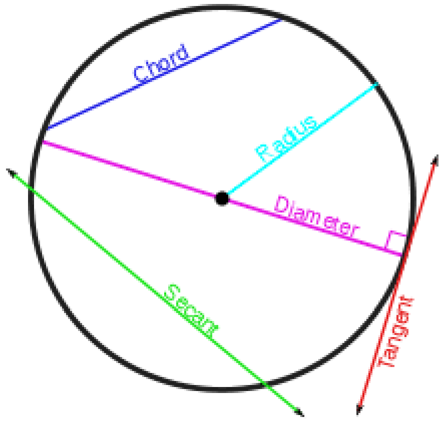
CIRCLES AND ITS CHORDS

CHORDS: A chord is a segment that joins two points of a circle

1. Chords are equidistant from the center if and only if their lengths are equal.
2. Equal chords are subtended by equal angles from the center of the circle.
3. A chord that passes through the center of a circle is called a diameter, and is the longest chord.
4. If the line extensions (secant lines) of chords AB and CD intersect at a point P, then their lengths satisfy $AP \cdot PB = CP \cdot PD$ (power of a point theorem).

TANGENTS:

Tangent to a circle is a line that touches the circle at one point, which is known as Tangency. At the point of Tangency, Tangent to a circle is always perpendicular to the radius.



Tangents Formula

we consider a circle where P is the exterior point. From that exterior point, the circle has the tangent at a points A and B. A straight line which cuts curve into two or more parts is known as a secant. So, here secant is PR is drawn and at Q, R intersects the circle as shown in the upper diagram. The formula for tangent-secant states that:

$$PR/PS = PS/PQ \Rightarrow PS^2 = PQ.PR$$

Properties of Tangents

Remember the following points about the properties of tangents-

- The tangent line never crosses the circle, it just touches the circle.
- At the point of tangency, it is perpendicular to the radius.
- A chord and tangent form an angle and this angle is same as that of tangent inscribed on the opposite side of the chord.
- From the same external point, the tangent segments to a circle are equal.

Theorems for Tangents to Circle

Theorem 1

A radius is obtained by joining the centre and the point of tangency. The tangent at a point on a circle is at right angles to this radius.

Theorem 2

This theorem states that if from one external point, two tangents are drawn to a circle then they have equal tangent segments. Tangent segment means line joining to the external point and the point of tangency.

TRIANGLES

Area of triangle (Hero's formula) = $\sqrt{(s-a)(s-b)(s-c)}$ S = $a+b+c2$

Based on sides Triangles are divided into

1 **Scaline Triangle:** No two sides are equal

2 **Isosceless Triangle:** Any two sides are equal

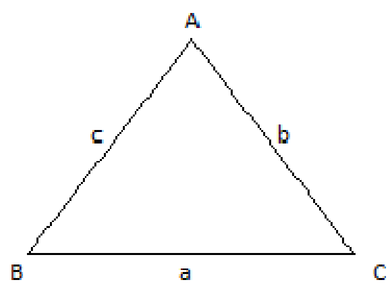
3 **Equilateral Triangle:** All sides are equal

Based on angles

1 **Acute angle Triangle:** The angle which is less than 900

2 **Right angle Triangle:** One angle should be 900

3 **Obtuse angle Triangle:** The angle which is greater than 900



In Triangle ABC, if 'a' is the largest side

$a^2 < b^2 + c^2$ => Acute angle Triangle

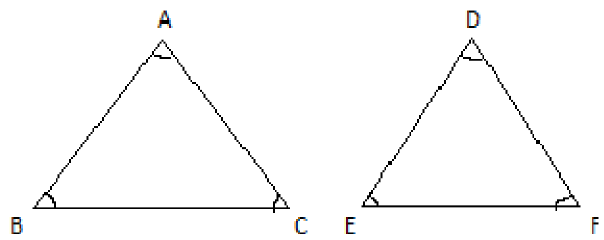
$a^2 = b^2 + c^2$ => Right angle Triangle

$a^2 > b^2 + c^2$ => Obtuse angle Triangle

Congruency of Triangle

Congruency means same (same in size and shape)

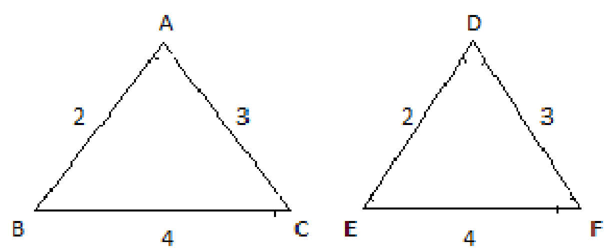
Three sides and three angles must be equal



The above two Triangles ABC & DEF have equal sides and equal angles Therefore,
$\Delta ABC \cong \Delta DEF$

1 SI**DE – SIDE – SIDE (SSS)**

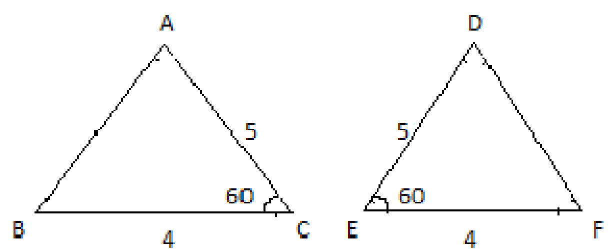
In both two Triangles all corresponding sides are equal



The above two Triangles, $\Delta ABC \cong DEF$

2 SIDE – ANGLE – SIDE (SAS)

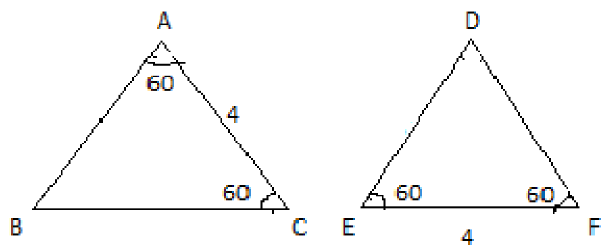
In this case, angle must be between two sides of Triangles



The above two Triangles, $\Delta BCA \cong \Delta FED$

3 ANGLE – SIDE – ANGLE (ASA)

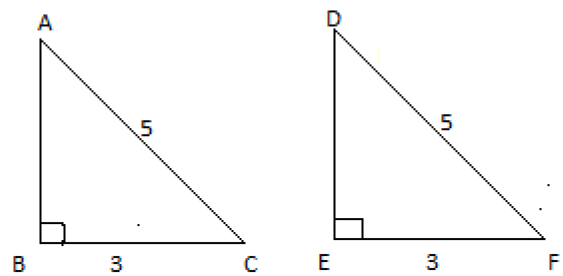
In this case, two angles and one side consider in both Triangles and side must be on angles side



So, $\Delta CAB \cong EFD$

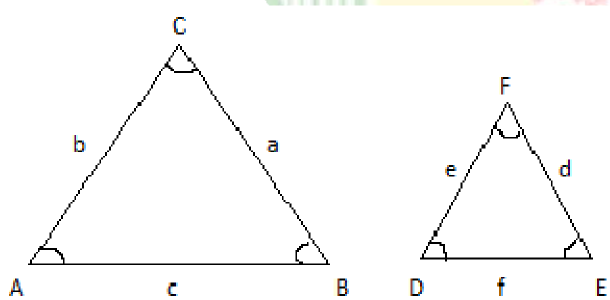
4 RIGHT ANGLE – HYPOTENUS – SIDE (RHS)

Here, in both the two Triangles, one angle must be 900 and Hypotenuse is same and one side is same



Similar Triangles

In case of Similar Triangles, shape is the same



From above two Triangles, angles are equal in both the Triangles and having same shape

So, $\Delta ABC \sim \Delta DEF$

AND

a/d= b/e= c/f= $h1/h2$= √Δ1√Δ2= $P1/P2$= $S1/S2$

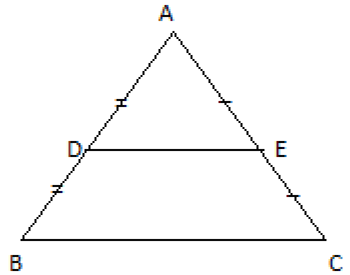
Here, h1 and h2 are the heights of both the Triangles ABC & DEF

Δ1 *and* Δ2 are the Areas of both the Triangles ABC & DEF

P1 and P2 are the Perimeters of both the Triangles ABC & DEF

S1 and S2 are the Semi – Perimeters of both the Triangles ABC & DEF

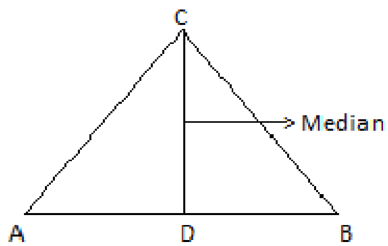
Mid – Point Theorem



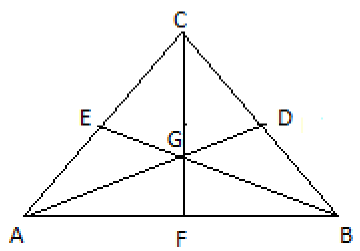
In Triangle ABC, D & E are the mid – points of AB & AC and DE is parallel to BC
Then, DE = $1/2 * BC$

MEDIANS & ITS PROPERTIES:

A line segment joining from the mid – point of the side to the opposite vertex



In Triangle ABC, DC is the median



Here, AB, BC & CA are sides
AD, BE & CF are Medians
G is Centroid

Centroid: The point of intersection of all the three medians of a Triangle is called Centroid

1 Median divides the Triangle into two equal parts

2 Centroid divides the Median in 2 : 1

3 Centroid always lies in inside the Triangle

4 Sum of the sides is greater than sum of the Medians AB + BC + CA > AD + BE + CF

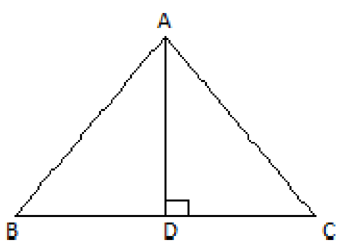
5 Formula for Median, $AD = 1/2 \times \sqrt{2AB^2 + 2AC^2 - BC^2}$

$BE = 1/2 \times \sqrt{2AB^2 + 2BC^2 - AC^2}$

$CF = 12 \times \sqrt{2AC^2 + 2BC^2 - AB^2}$

ALTITUDE & ITS PROPERTIES

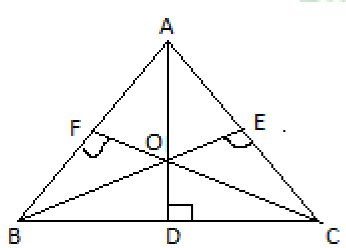
The Altitude of a Triangle is a line segment perpendicularly drawn from vertex to the side opposite to it. The side on which the perpendicular is being drawn is called its base



In Triangle ABC,

AB, BC & CA are sides

AD = altitude



In Triangle ABC
AB, BC & CA are sides
AD, BE & CF are altitudes
O = Ortho – centre

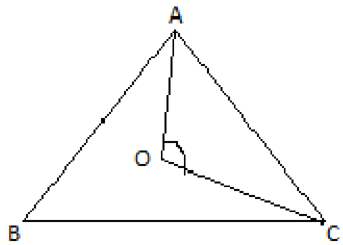
ORTHO – CENTRE The point of intersection of all the three Altitudes of a Triangle is called its Ortho – centre.

1 Sum of the sides of Triangle is greater than sum of the Altitudes AB + BC + CA > AD + BE + CF

2 In Acute angleTriangle, Orho – centre lies in inside the Triangle

3 In Right angle Triangle, Ortho – centre lies on the right angle

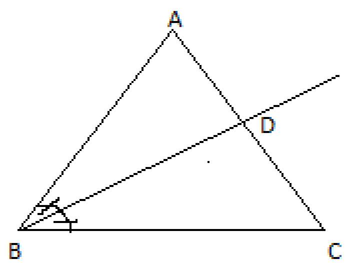
4 In Obtuse angle Triangle, Ortho – centre lies on the outside the Triangle



Here, O = Ortho – centre
Then, $\angle AOC = 180° - \angle B$
AND, $\angle BOC = 180° - \angle A$ $\angle AOB = 180° - \angle C$

ANGLE BI – SECTOR & ITS PROPERTIES

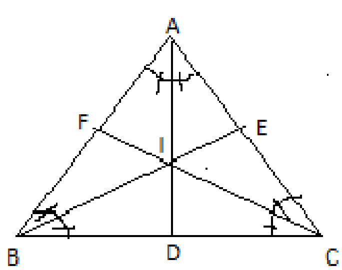
The Angle Bi – sector is, the line which is bi – sects the angle equally (divided into two equal parts)



Here

AB, BC & CA are sides of Triangle

BD = angle bi – sector line

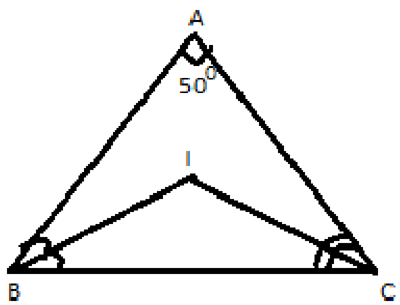


In Triangle ABC
AB, BC & CA are sides
AD, BE & CF are Angle bi – sector lines

I = In – centre of a Trianlge

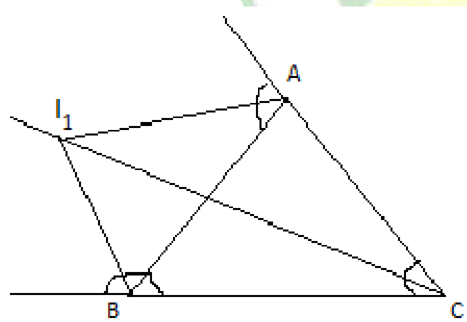
IN – CENTRE:
∟BIC = 90°+ ∟$A/2$
∟CIA = 90°+ ∟$B/2$
∟AIB = 90° + ∟$C/2$

EXAMPLE: ABC is a Triangle BI and CI are angle bisectors of ∟ABC and ∟ACB is ∟BAC = 500, find ∟BIC?
SOLUTION:



SHORT METHOD:
∟BIC = 90°+ 1/2∗ ∟A [Theorem] ∟BIC = 115°

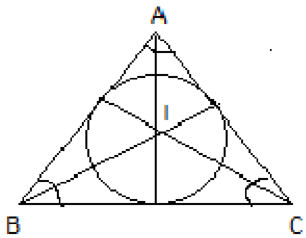
EX – CENTRE

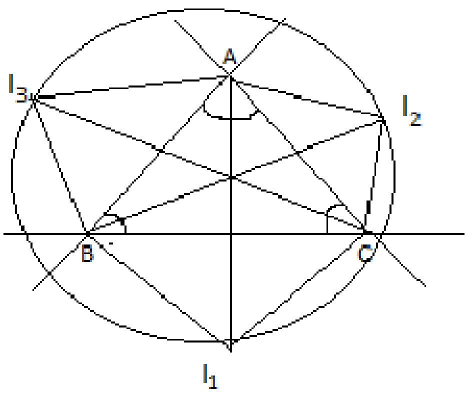


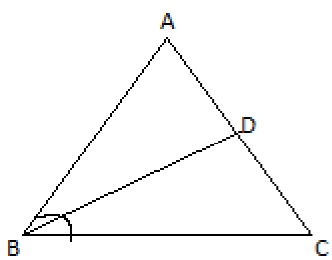
Ex – centre is formed by two external angles bi – sector and one internal angle bi – sector.

1 There is only one in – centre
2 And, there are three Ex – centre formed
3 In – centre is equidistance from the sides

IN – CIRCLE

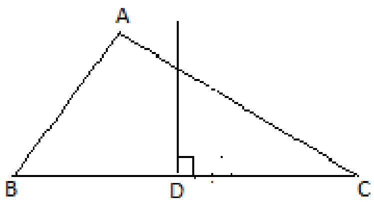






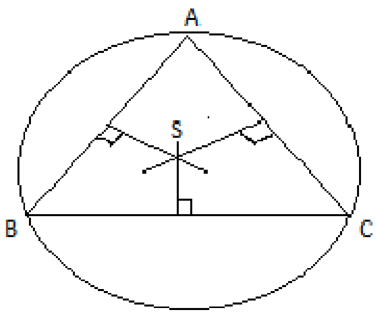
BD = Angle of bi – sector
$BC/BA = CD/DA$

PERPENDICULAR BI – SECTOR & ITS PROPERTIES
Perpendicular bi – sector is, a line passes through the mid – point of opposite vertex perpendicularly



In Triangle ABC, D is the mid – point of BC

CIRCUM – CENTRE

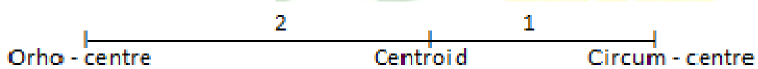


In Triangle ABC,
S = Circum – centre
The circle is formed is Circum – circle
1 Circum – centre is equidistance from all the vertices

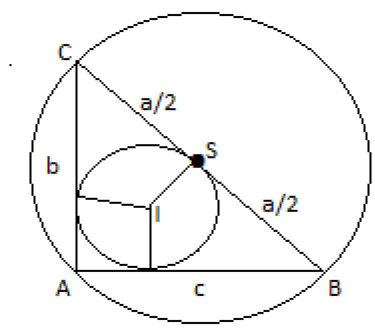
2 In Acute angle Triangle, circum – centre lies in inside the Triangle

3 In Right angle Triangle, circum – centre lies on mid – point of Hypotenuse

4 In Obtuse angle Triangle, circum – centre lies on outside the Triangle

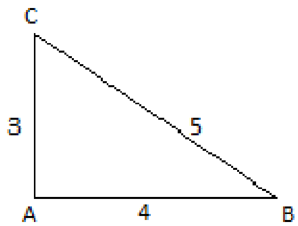


RIGHT ANGLE TRIANGLE



In Right angle Triangle ABC
I = In – centre
S = Circum – centre
In – radius (r) = $b+c-a/2$
Circum – radius (R) = $a/2$
Area of Right angle Triangle = $1/2*base*height$

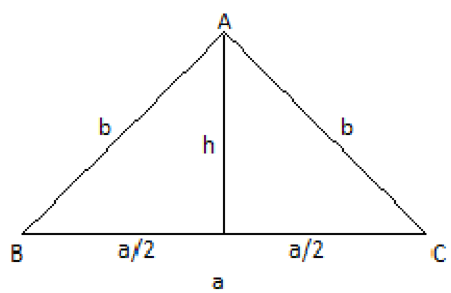
PYTHOGOROS TRIPLEX



3, 4 & 5 => $3^2 + 4^2 = 5^2$
5, 12 & 13 => $5^2 + 12^2 = 13^2$
7, 24 & 25 => $7^2 + 24^2 = 25^2$
8, 15 & 17 => $8^2 + 15^2 = 17^2$
9, 40 & 41 => $9^2 + 40^2 = 41^2$
11, 60 & 61 => $11^2 + 60^2 = 61^2$

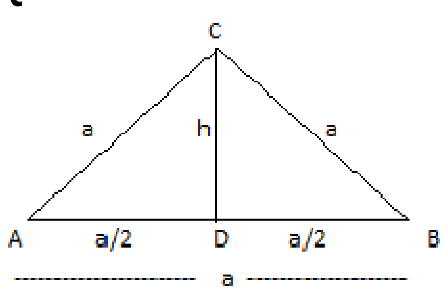
ISOSCELES TRIANGLE

The Triangle which have two equal sides and other is different



Here,
AB = AC
∟B = ∟C
$\Delta ABD \cong \Delta ACD$
h = D is the mid – point of BC then, BD = DC

EQUILATERAL TRIANGLE



In Equilateral Triangle, all sides and all angles are equal
Here, AB = BC = CA = a
∟A = ∟B = ∟C = 60°

h = height

From diagram, $a^2 = h^2 + (a/2)^2$

By solving, we get $h = \sqrt{3}/4 \times a$

Area = $1/2 \times a \times h \Rightarrow 1/2 \times a \times \sqrt{3}/4\ a$

$= \sqrt{3}/4 \times a^2$

In – radius (r) = $a/2\sqrt{3}$

Circum – radius (R) = $a/\sqrt{3}$

In Equilateral Triangle Centroid, Ortho – centre, In – centre and Circum – centre are coincide at the same point

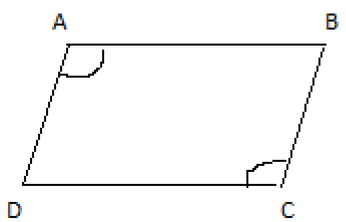
QUADRILATERALS

Sum of the angles in a Quadrilateral is 360°

Quadrilateral means contain four sides

PARALLELOGRAM

In Parallelogram, opposite sides are equal and opposite angles are equal



Here, AB //DC

BC//AD

AB = DC & BC = AD

Opposite angles are equal, ∟A = ∟C & ∟B = ∟D

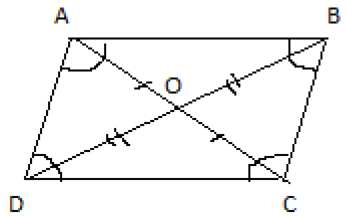
Adjacent angles sum = 180°

∟A + ∟B = 180°

∟B + ∟C = 180°

∟C + ∟D = 180°

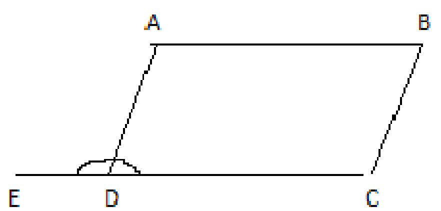
∟D + ∟A = 180°



Diagonals bi – sect each other, OA = OC & OB = OD
Here, AC & BD are diagonals
Diagonals divide the Parallelogram into four equal areas

$\Delta AOB \cong \Delta COD \cong \Delta AOD \cong \Delta BOC$

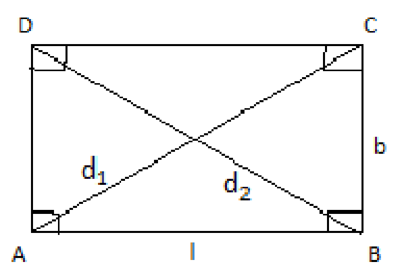
Sum of the squares of the sides = Sum of the squares of the Diagonals
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$



If AB//DC, then CDE is Transversal

RECTANGLE

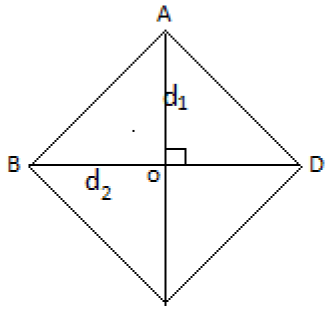
Rectangle has two equal opposite sides and all angles are same with 90°



Here, AB = DC
AD = BC
l = length
= breadth
d1 & d2 = diagonals
Area = l × b
Perimeter = 2(l +b)

Diagonal = $\sqrt{l2 + b2}$

RHOMBUS



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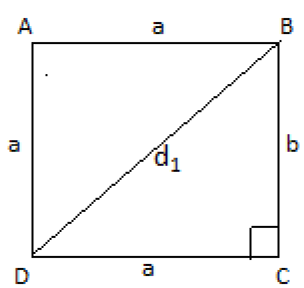
All sides are equal
Diagonals are perpendicular to each other
Diagonals bi – sect the interior angles
AB = BC = CD = DA

d1 & d2 = diagonals

Side = $\frac{1}{2} \times \sqrt{d_1^2 + d_2^2}$
Area = $\frac{1}{2}$ × (d1 × d2)

Perimeter = $2\sqrt{d_1^2 + d_2^2}$
The four Triangles formed are congruent to each other
$\Delta AOB \cong \Delta AOD \cong \Delta BOC \cong DOC$

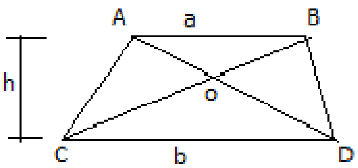
SQUARE



AB = BC = CD = DA = a = side
d = diagonal
Area = a^2

Perimeter = 4a
Diagonal = $\sqrt{2} \times a$

TRAPEZIUM



Here, AB// CD
a, b are the lengths of the parallel sides
h = height between the parallel lines
Area = $1/2 \times h \times (a+b)$
Median = $\mathbf{1/2} \times (a + b)$
Area of $\Delta AOC = Area\ of\ \Delta BOD$
Perimeter = AB + BC + CD + DA

EXAMPLE: The parallel sides of a Trapezium are in a ratio 2 : 3 and their shortest distance is 12 cm. If the area of the Trapezium is 480 sq. cm., the longer of the parallel sides is of length?

SOLUTION:

Sides of the Trapezium (l1 & l2) = 2x and 3x cm

Height (shortest distance), h = 12 cm

Area = $\mathbf{1/2} \times$ (l1 + l2) × h

Here, l1 & l2 are parallel sides
h = distance between the , l1 & l2
$\therefore 480 = 1/2 \times (2x+3x) \times 12$
x = 16
Therefore, longer side = 3x => 3 × 16
= 48 cm

POLYGONS

Polygon is nothing but contains more than four sides
In Regular Polygon = All interior angles
Sum of interior angles of any Polygon = $(n - 2) \times 180°$
No. of Diagonals of Polygon = $n \times (n-3)/2$

n = no. of sides

REGULAR POLYGON

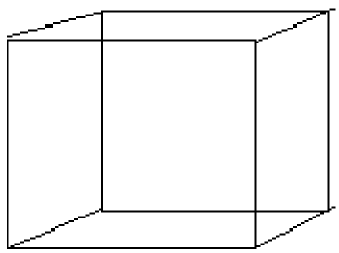
All interior angles are equal

Each exterior angle = $360°/n$

Each interior angle = $(n-2) \times 180°/n$

Exterior angle = 180° – interior angle

CUBE



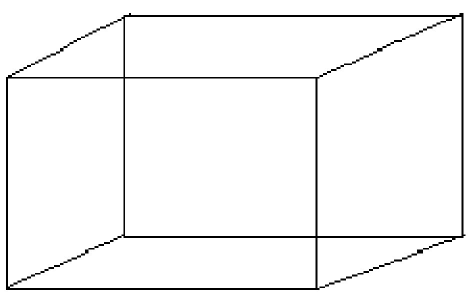
Volume = a^3 (a = side of cube)

Lateral surface area = $4 \times a^2$

Total surface area = $6 \times a^2$

Diagonal = $\sqrt{3} \times a$

CUBOID



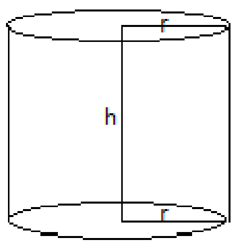
Volume = length (l) × breadth (b) × height (h)

Total surface area = 2× (lb + bh + lh)

Area of four walls of a room = 2h (l + b)

Diagonal = $\sqrt{l^2 + b^2 + h^2}$

CYLINDER



Here, r = radius, h = height

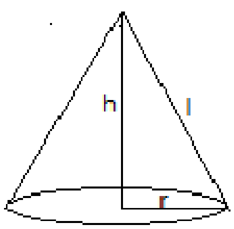
Volume (v) = $\pi r^2 h$

Curved surface area = $2\pi rh$

Total surface area = $2\pi rh + 2\pi r^2$

Cylinder is nothing but no. of circles placed one by one vertically

CONE



Here, r = radius, h = height

$l = \sqrt{r^2 + h^2}$

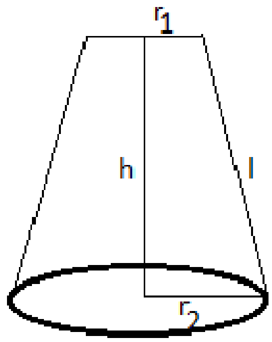
Volume (v) = $1/3\, \pi r^2 h$

Curved surface area = πrl

Total surface area = $\pi rl + \pi r^2$

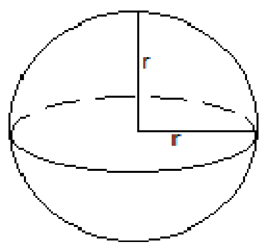
Cone is formed by, when Cylinder is divided into three parts

Frustum of Cone



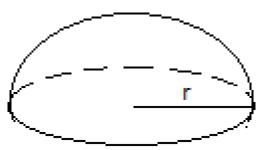
Here, r1 & r2 are radii of frustum
h= height, l = length
Then, = $\mathbf{1/3} \times \boldsymbol{\pi}$ ($r1^2$ + $r2^2$ + r1 × r2) × h
Surface area = $\boldsymbol{\pi}$ ×l (r1 + r2) + $\boldsymbol{\pi} r1^2$ + $\boldsymbol{\pi} r2^2$

SPHERE



Here, r = radius
Volume (v) = $\mathbf{43}\ \pi r^3$
Total surface area = 4 πr^2

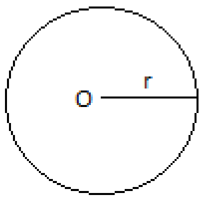
HEMISPHERE



Here, r = radius
Spherical Cell

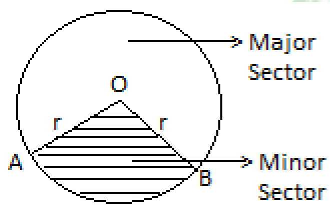
Volume = $\mathbf{4/3} \times \boldsymbol{\pi}$ ($R^3 - r^3$)
Total surface area = 4 × $\boldsymbol{\pi}$ ($R^2 - r^2$)

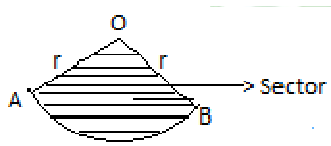
CIRCLES



Here, r = radius O = Centre
Area = πr^2
Perimeter = $2\pi r$

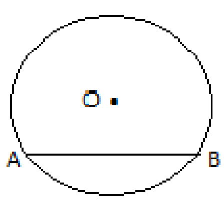
FOR SEMICIRCLE
Area = $\pi r^2/2$
Perimeter = $\pi r + 2r$ or $\boldsymbol{r} \times \mathbf{36/7}$
Area of the ring or circular path = $\boldsymbol{\pi}\,(R^2 - r^2)$





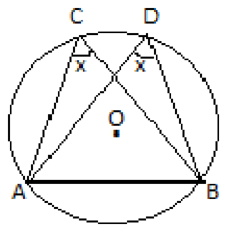
Length of the arc
= $2\pi r \times \boldsymbol{\theta/360}$
Area of sector
= $\pi r^2 \times \boldsymbol{\theta/360}$
Perimeter of sector
= $2\pi r \times \boldsymbol{\theta/360} + 2r$

CHORD:

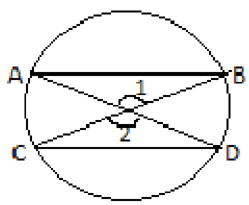


Here, O = Centre
AB = Chord

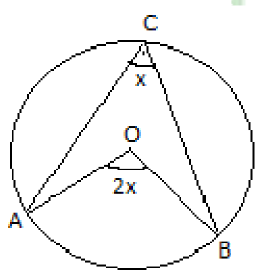
largest chord is diameter



Here, AB = Chord
O = Centre
From Chord to any point on circle joins the lines, angle occurred at different places is same
∟x = ∟x

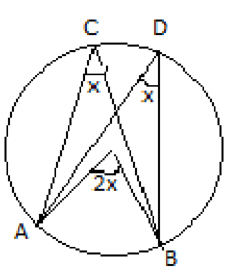


Here, AB & CD are two equal chords (AB = CD), then **<1** = <2



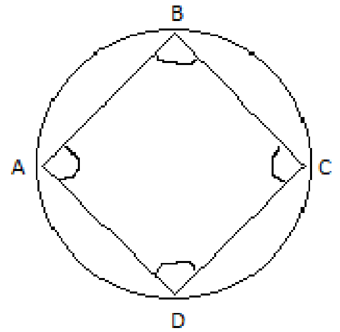
The angle subtended by an arc of circle at the centre is double the angle subtended by it at any point on the remaining part of the circle

i. e <AOB = 2×<ACB

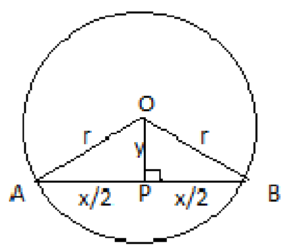


Angle in the same segment of a circle is equal
<AOB = 2× <ACB

<AOB =2× <ADB
<ACB = <ADB

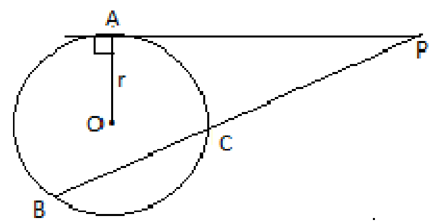


The opposite pairs of angles of cyclic quadrilateral is supplementary each other
<A + <C = 180°
<B + <D = 180°



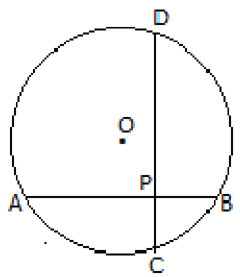
From above diagram,
AB = Chord
O = Centre
P = Mid point of A & B
r = radius
Then, $r^\circ = y^\circ + (x/2)$

TANGENTS & SECANTS

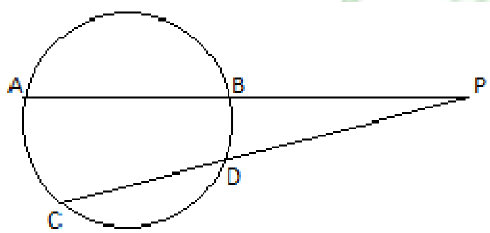


i. Tangent touch the circle at one point while secant touch the circle at two points Here, AP = Tangent

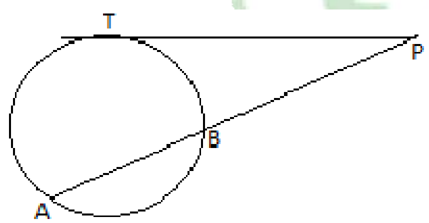
BCP = Secant and <A = 90° always



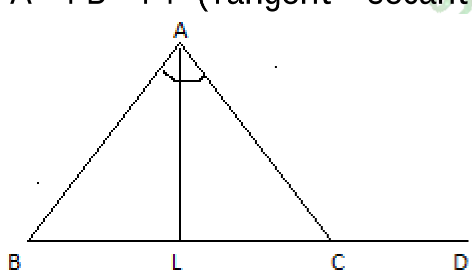
If two chords AB and CD intersect internally at a point P
PA × PB = PC × PD



If two chords AB & CD intersect externally at a point P. Then
PA × PB = PC × PD

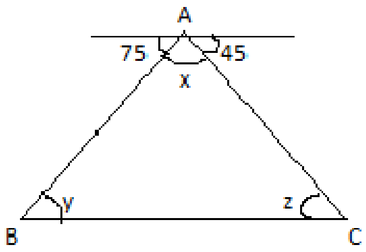


If ABP is secant to a circle intersecting the circle at A & B, and PT is tangent, then
PA × PB = PT^2 (Tangent – Secant theorem)



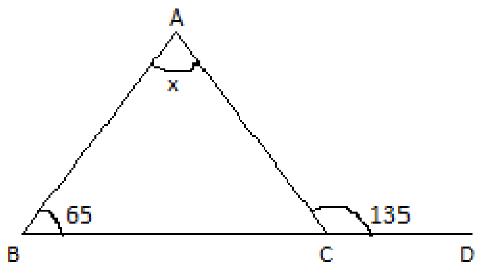
The side of BC of ΔABC is produced to D
The bisector of LA meets BC at L, then

i. <ABC +<ACD = 2× <ALC



Here, <Y = <75 & <Z = <45 (Alternate angles are equal)

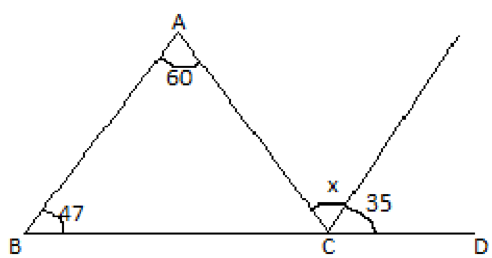
∴ <X = 60°[180 – (75+45)]



Here, external angle = 135°

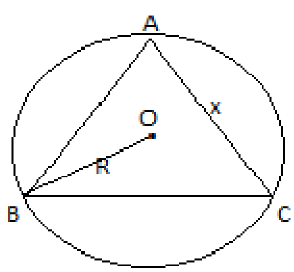
And, 135° = X + 65°(<A + <B = 135)

X = 70°



Here, X + 35°= 60°+ 47°

X = 72°

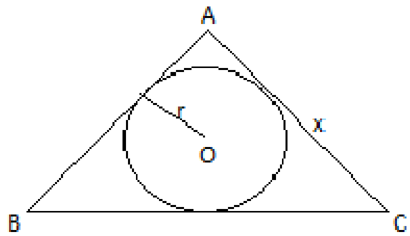


Here, R = circum radius

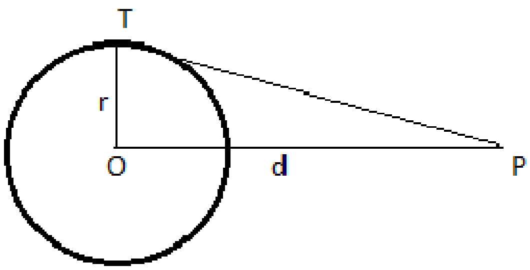
X = side of a equilateral triangle

O = centre of circle When an equilateral triangle is inscribed in a circle

Radius (R) = $x/\sqrt{3}$

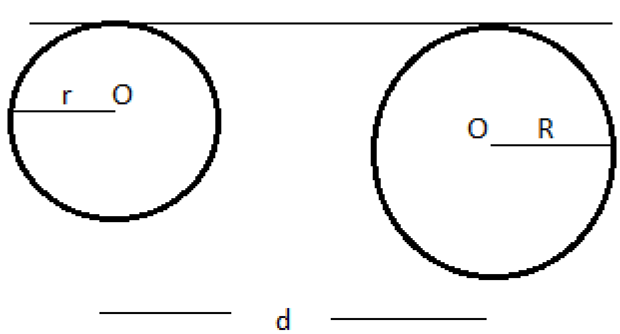


Here, r = in-radius
X = side of a equilateral triangle
O = centre of circle
When a circle is inscribed in an equilateral triangle Radius (r) = $x/2\sqrt{3}$



Here, r = radius
O = centre of circle
T = tangent
P = external point from center
And, d = distance between the center of circle and external point
Then, length of the Tangent = $\sqrt{d^2 - r^2}$

Direct Common Tangent



If the two circles are on the same side of a line, the common tangent is said to be direct common tangent

Here, r = radius of one circle

R = radius of another circle

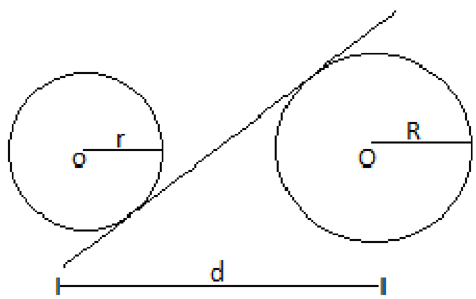
O = centre of circle And,

d = distance between two centers of circles

Then, the length of the direct common tangent = $\sqrt{\boldsymbol{d2} - (\boldsymbol{R} - \boldsymbol{r})}$

Transverse Common Tangent

If the two circles are on the opposite side of a line, the common tangent is said to be transverse common tangent



Here, r = radius one circle

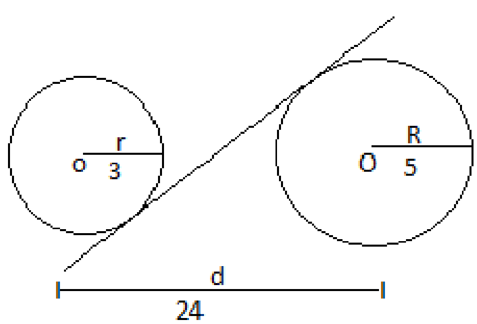
R = radius of another circle

O = center of circles

And, d = distance between the centers

Then, the length of the Transverse common tangent = $\sqrt{\boldsymbol{d}^2 - (\boldsymbol{R} + \boldsymbol{r})}$

EXAMPLE: The radii of two circles are 5 cm and 3 cm respectively and the distance between their centres is 24 cm. Then the length of the Transverse common tangent is ?



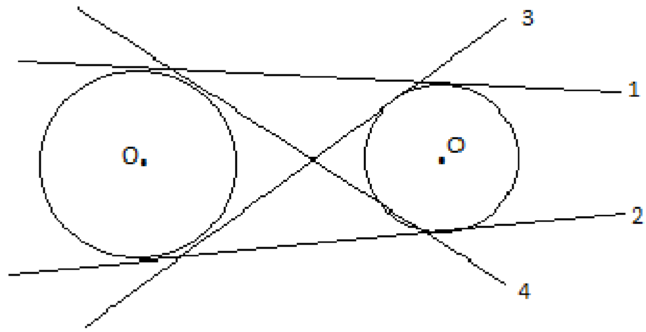
SOLUTION:

Transverse common tangent = $\sqrt{\boldsymbol{d}^2 - (\boldsymbol{R} + \boldsymbol{r})}$

Here, d = 24 cm, r = 3 cm and R = 5 cm

$= \sqrt{24^2 - (5+3)^2}$

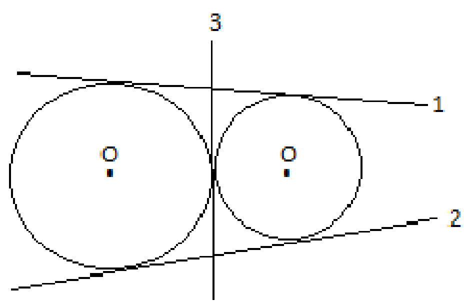
$= 16\sqrt{2}$ cm



Here, O = Centre

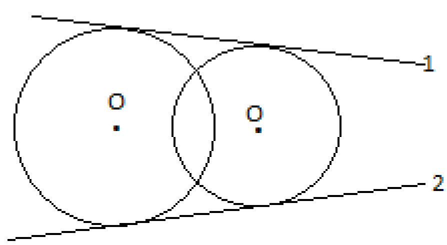
1 & 2 are Direct Common Tangents

3 & 4 are Transverse Common Tangents



Here, O = Centre

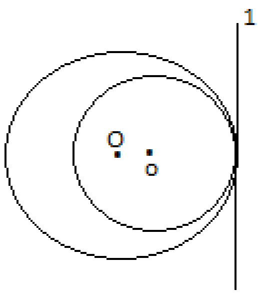
1 & 2 are Direct Common Tangent and 3 is a Transverse Common Tangent



Here, O = Centre

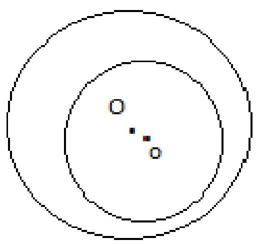
1 & 2 are Direct Common Tangent

Transverse Common Tangent = 0



Here, 1 is a Direct Common Tangent

Transverse Common Tangent = 0



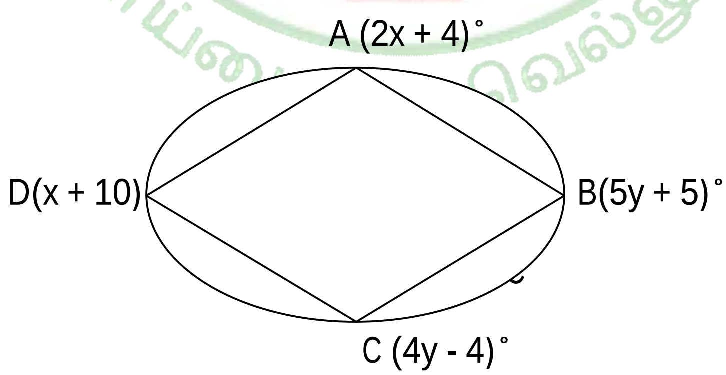
Here, O = Centre

Direct Common Tangent = 0

Transverse Common Tangent = 0

PROBLEMS:

1. **In the following figure, Find the value of y.**



a) 20˚ b) 10˚ c) 25˚ d) 30˚

Soln:

Since, ABCD is a cyclic quadrilateral

$A + C = 180^\circ$

$2x + 4 + 4y - 4 = 180^\circ$

(sum of opposite angles of a cyclic quadrilateral in 180°)

$x + 2y = 90^\circ$

$B + D = 180^\circ$

$5y + 5 + x + 10 = 180^\circ$

$x + 5y = 165^\circ$

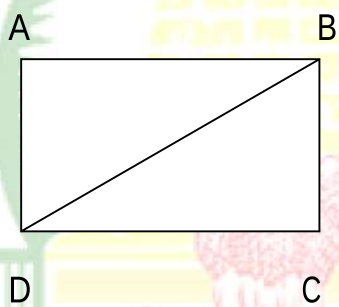
$3y = 75^\circ$

$y = 75^\circ/3$

$y = 25^\circ$

Ans: c) 25°

2. If ABCD is a parallelogram, then find the value of x and y.



a) $4^\circ, 5^\circ$ b) $5^\circ, 4^\circ$ c) $6^\circ, 5^\circ$ d) None of the above

Soln:

AB II DC

ABD = BDC

$28^\circ = 7y$

$y = 28^\circ/7$

$y = 4^\circ$

similarly, AD II BC

ADB = DBC

$12x = 60^\circ$

$x = 60^\circ/12$

$x = 5^\circ$

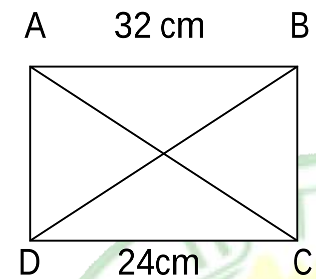
Ans: 5˚, 4˚

3. **In a rectangle ABCD, diagonal AC and BD intersect each other at O. If AB = 32 cm and AD = 24 cm, find the length of OD?**

   a) 40 cm b) 60 cm c) 80 cm d) 20 cm

   Soln:

   Clearly BAD is a right angled triangle,



$(BD)^2 = (AB)^2 + (AD)^2$

$= (32)^2 + (24)^2$

$= 1024 + 576$

$= 1600$

BD = 40 cm

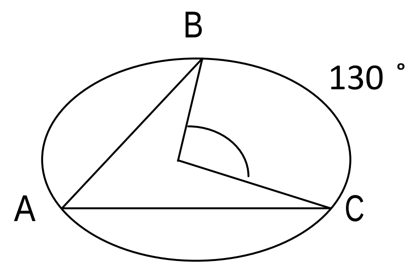
OD = BD/2

= 40/2

= 20 cm

Ans: d)20cm

4. **In the adjoining figure, O is the centre of the circle. Find the value of BAC?**



a) 65˚ b) 90˚ c) 80˚ d) 45˚

Soln:

BAC = ½ BOC

= ½ × 130˚

= 65˚

Ans: a)65°

5. **In a rhombus, ABCD B = 60˚, AB = 14 cm. Find the diagonal AC?**

a) 7 cm b) 14 cm c) 14√2 cm d) cannot be determined

Soln:

ADC = ½ × AOC

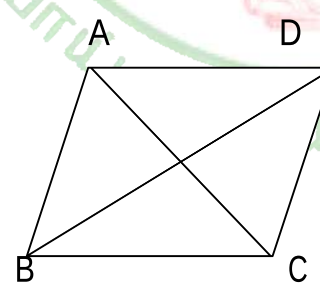
= ½ × 140˚

= 70˚

ABCD is rhombus

AB =BC

BAC = BCA



Now, in ABC,

BAC + ABC + BCA = 180˚

BAC + 60˚ + BAC = 180˚

(BAC = BCA)

2BAC = 120

BAC = 120/2

= 60˚

ABC = BAC = BCA = 60˚

ABC is equilateral,

Thus, AC = AB = 14 cm

Ans: b) 14 cm

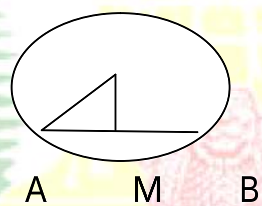
6. **From the centre of a circle, distance of a chord is 16 cm. If radius of the circle be 20 cm. What will be the length of the chord?**

   a) 6 cm b) 12cm c) 24 cm d) 36 cm

   Soln:

   According to the question,

   OM = 16 cm



In right angled OMA

$(OA)^2 = (OM)^2 + (AM)^2$

$(AM)^2 = (OA)^2 – (OM)^2$

$(AM)^2 = (20)^2 - (16)^2$

= 400 – 256

= 144

AM = 12 cm

Thus, the length of chord,

AB = 2,

AM = 2 × 12

= 24 cm

ANS: c) 24 cm

7. **In a cyclic quadrilateral ABCD A= 4x˚, B = 7x˚, c = 5y˚ and d = y˚. Find the ratio y : x?**
   a) 2 : 3 b) 3 : 2 c) 3 : 4 d) 4 : 3

   Soln:

   In cyclic quadrilateral ABCD

8. **If (3x + 15)˚ and (x + 5)˚ are supplementary angles, find the value of x?**
   a) 20˚ b) 30˚ c) 40˚ d) 60˚

   Soln:

   (3x+15)˚ and (x + 5)˚ are supplementary,

   (3x + 15)˚ + (x + 5)˚ = 180 ˚

   4x + 20˚ = 180˚

   4x = 180˚ - 20˚

   4x = 160˚

   x = 160˚/4

   x = 40˚

   Ans: c) 40˚

9. **The ratio of angles of a triangle are in the ratio of 2 : 3 : 4. Find the largest angle of the triangle?**
   a) 20˚ b) 40˚ c) 60˚ d) 80˚

   Soln:

   Let angles of the triangle are 2x, 3x and 4x, respectively, then

   2x + 3x + 4x = 180˚

   9x = 180˚

   x = 180˚/9

   x = 20˚

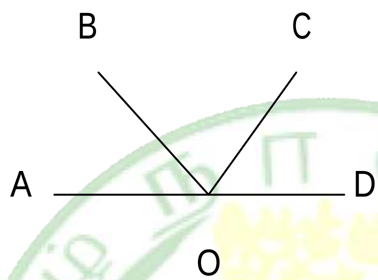
   Thus, the largest angle = 4x

   = 4 × 20˚

= 80˚

Ans: d) 80˚

10.Find the value of z.



a) 120˚ b) 240˚ c) 180˚ d) 360˚

Soln:

z/3 + z/4 + z/6 = 180˚

9z/12 = 180˚

z = 180˚ × 12/9

= 240˚

Ans: b) 240˚

11.An angle is greater than it's supplementary angle by 20˚. Find the angle?

a) 80˚ b) 90˚ c) 100˚ d) 120˚

Soln:

Let the angle be x˚

Then, its supplementary angle = (180˚ - x)

According to the question,

x – (180˚- x) = 20˚

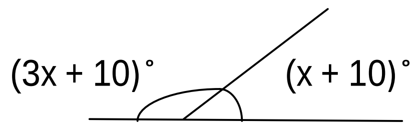
2x – 180˚ = 20˚

2x = 200˚

x = 100˚

ANS: c) 100˚

12.In the following figure, find the value of x?



a) 20˚ b) 30˚ c) 40˚ d) 50˚

Soln:

According to the figure,

(3x+10)˚ + (x + 10)˚ = 180˚

4x + 20˚=180˚

4x =180˚- 20˚

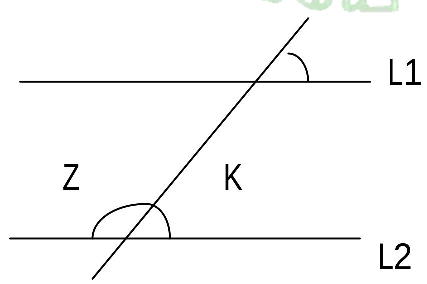
4x = 160˚

x = 160˚/4

x = 40 ˚

Ans: c) 40˚

13.In the adjoining figure, find the value of k.



a) 40˚ b) 160˚ c) 80˚ d) 120˚

Soln:

Since, L1 and L2 are parallel lines,

therefore 40 = k

Ans: a) 40˚

14.In the above question, find the value of Z?

a) 140˚ b) 120˚ c) 80˚ d) 120˚

Soln:

From the figure, Z + K = 180˚

Z = 180˚ - k

= 180˚- 40˚

= 140˚

Ans: a) 140˚

15.In the adjoining figure, if BAC=55˚ and the triangleis isosceles, then find the value of B?

a) 72˚ b) 62˚ c) 60˚ d) None of these

Soln:

ABC is isosceles with AB = AC

ABC = ACB

BAC + ABC + ABC = 180˚

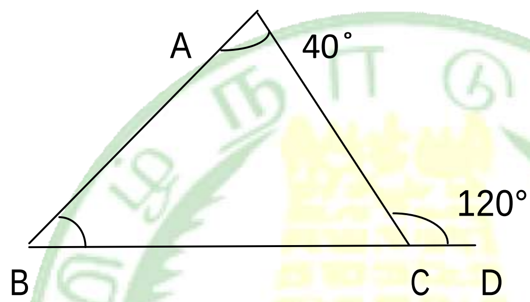
(ABC = ACB)

2ABC = 125

ABC = 62.5

Ans: d) None of these

16.What will be the value of X in adjoin figure.



a) 50˚ b) 80˚ c) 70˚ d) None of these

Soln:

ACD is an external angle of ABC

ACD = ABC + BAC

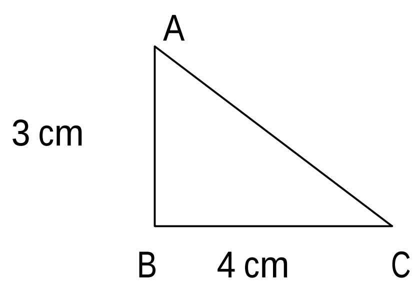
$120^\circ = x + 40^\circ$

$x = 120^\circ - 40^\circ$

$x = 80^\circ$

Ans: b) 80˚

17.In the adjoining figure, if B = 90˚, find the length of AC.



a) 6m b) 5m c) 7 m d) 8 m

Soln:

Since, B = 90˚

By Pythagoras theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (3)^2 + (4)^2$

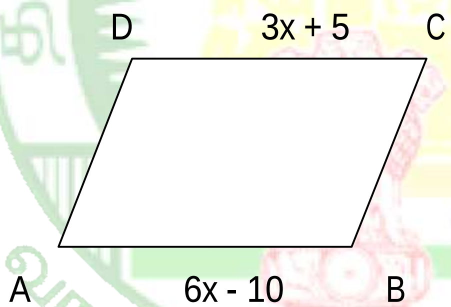
$= 9 + 16$

$= 25$

AC = 5 cm

Ans: b) 5m

18.Find the value of AD in the parallelogram ABCD.



a) 20 units b) 45 units c) 30 units d) None of these

Soln:

ABCD is a parallelogram,

AB = DC

$6x – 10 = 3x + 5$

$6x – 3x = 5 + 10$

$3x = 15$

$x = 15/3$

$x = 5$

Now, AD=BC=4x-5

$= 4 \times 5 – 5$

= 20 – 5
= 15

Ans: d) None of these

19. In three angles of a quadrilateral are in the ratio 3 : 2 : 1 and the fourth angle is equal to these three, then find the value of smallest angle?

a) 15˚ b) 20˚ c) 30˚ d) 40˚

Soln:

Let three angles of the quadrilateral are 3x, 2x and x respectively.

Then, fourth angle

= 3x + 2x + x = 6x

Now, sum of all the four angles = 360˚

$3x + 2x + x + 6x = 360^\circ$

$12x = 360^\circ$

$x = 360^\circ/12$

$x = 30^\circ$

Thus, smallest angle = 30˚

Ans: 30˚

20. If MNOP be a cyclic quadrilateral with M : N: O = 1:3:4, then find the value of P?

a) 108˚ b) 36˚ c) 72˚ d) 144˚

Soln:

Let M = x, N = 3x, O = 4x

MNOP is cyclic quadrilateral

M + O = 180˚

$x + 4x = 180^\circ$

$5x = 180^\circ$

$x = 180^\circ/5$

$x = 36^\circ$

N = 3x = 3 × 36˚

$= 108^\circ$

$N + P = 180^\circ$

$P = 180^\circ - N = 180^\circ - 108^\circ$

$= 72^\circ$

Ans: 72°